Time - Evolving Signal Analysis

Roza ACESKA
Ball State University

Midwestern Workshop on Asymptotic Analysis
Indiana U. Purdue U. Fort Wayne, 2016

Introduction

- The problem of recovering an evolving signal from a set of samples taken at different time instances is motivated by research questions emerging in dynamical systems.

  Lu, Veterli. *Spatial super-resolution of a diffusion fieldby temporal oversampling in sensing networks.* IEEE Int. Conf. Acoustics, Speech and Signal Proc. 2009.

- Sampling problems in dynamical systems: eg. sampling of air pollution, wireless networks, temperature distribution over a metropolitan area etc.

- We state the problem of spatio-temporal sampling for different classes of functions (signals), and provide specific reconstruction results.

What is Dynamical Sampling?

Let f describe the initial state of a physical system on domain D. Over time, the system evolves to the state

$$f_t = A_t f,$$

where $\{A_t\}_{t \geq 0}$ is a family of evolution operators.

- at time instances t_i: sampling sets $X_i \subset D$
- S_{X_i} - corresponding downsampling operator
  (obtains insufficiently many samples for 'successful' recovery)

The fundamental question in dynamical sampling

Is the recovery of the initial state f possible from the coarsely under-sampled initial state and its altered states $f_t = A_t f$ at time instances $\{t_i : i = 1, \cdots, L\}$?

Goal: Reconstruct $f = f_0$ from

$$S_{X_0}(f), S_{X_1}(A_{t_1} f), ..., S_{X_L}(A_{t_L} f). \quad (1)$$

Conditions ...

... on evolution operators $A_i = A_{t_i}$, sampling sets X_i, number L of repeated subsampling harvests, and sampling time instances t_i

Recovery of the initial state $f = f_0$ is possible, if we have:

- IS *Invertibility sampling property*. Within a class of signals, any signal h is associated with a samples data set $\{S_{X_i}(A_i h_i)\}$ which uniquely determines h.
- SS *Stability sampling property.* Within a class of signals, given any two signals h, $\tilde{h}$, the following two norms,

$$\|h - \tilde{h}\|_p^2 \text{ and } \sum_{i=0}^{L} \|S_{X_i} A_i (h - \tilde{h})\|_{\ell^p}^2 \text{ are equivalent.}$$

BALL STATE UNIVERSITY.

Convolution operators in Shift-Invariant Spaces (I)

Example: If $\operatorname{supp} \hat{f} \in [-1/2, 1/2]$ and $f \in L^1 \cap L^2(\mathbb{R})$, then

$$\text{(Shannon Sampling Thm)} \qquad f(x) = \sum_k f(k) sinc(x-k).$$

Let $\varphi \in \{f | \sum_{k\in\mathbb{Z}} \sup_{0\le x\le 1} |f(x+k)| < \infty\} \cap C$.

Shift-Invariant space

$$V(\varphi) := \{c * \varphi = \sum_{k\in\mathbb{Z}} c_k \varphi(\cdot - k) \mid c = (c_k) \in \ell^2(\mathbb{Z})\} \qquad (2)$$

- If $0 < M_1 \le \sum_{k\in\mathbb{Z}} |\hat{\varphi}(\xi + k)|^2 \le M_2 < \infty$, then every $f \in V(\varphi)$ can be recovered from the samples $f(\mathbb{Z})$.

Question: Let $f = f_0 \in V(\varphi)$ and $f_j = a * \cdots * a * f$; can we recover f from subsamples $\{f(m\mathbb{Z}), f_1(m\mathbb{Z}), f_2(m\mathbb{Z}), \dots\}$?

Answer: Yes.

Fact: Let $\Phi_j = \varphi_j|_{\mathbb{Z}}$, where $\varphi_j = a^j * \varphi$, $0 \le j \le L$. Then

$$\hat{\Phi}_j \in C([0,1]).$$

Theorem

Every $f \in V(\varphi)$ can be recovered in a stable way from the measurements $y_j = (a^j * f)|_{m\mathbb{Z}}$, $0 \le j \le m-1$, if and only if $\det \mathcal{A}_m(\xi) \ne 0$ for all $\xi \in [0,1]$, where

$$\mathcal{A}_m(\xi) = \begin{pmatrix} \hat{\Phi}_0(\frac{\xi}{m}) & \hat{\Phi}_0(\frac{\xi+1}{m}) & \dots & \hat{\Phi}_0(\frac{\xi+m-1}{m}) \\ \hat{\Phi}_1(\frac{\xi}{m}) & \hat{\Phi}_1(\frac{\xi+1}{m}) & \dots & \hat{\Phi}_1(\frac{\xi+m-1}{m}) \\ \vdots & \vdots & \vdots & \vdots \\ \hat{\Phi}_{m-1}(\frac{\xi}{m}) & \hat{\Phi}_{m-1}(\frac{\xi+1}{m}) & \dots & \hat{\Phi}_{m-1}(\frac{\xi+m-1}{m}) \end{pmatrix}.$$

Proof:

Due to $$\mathcal{F}(f_j|_{m\mathbb{Z}})(\xi) = \frac{1}{m}\sum_{l=0}^{m-1} \hat{c}(\frac{\xi+l}{m})\hat{\Phi}_j(\frac{\xi+l}{m}). \qquad (3)$$

Special case: $A_i = A$ in a separable Hilbert space I

Let $\mathcal{H}$ be a separable (complex) Hilbert space, $f \in \mathcal{H}$ be an unknown vector and $f_n \in \mathcal{H}$ be the state of the system at time n.

We assume:

$$f_0 = f, \quad f_n = Af_{n-1} = A^n f$$

where A is a known bounded operator on $\mathcal{H}$.

Given the measurements:

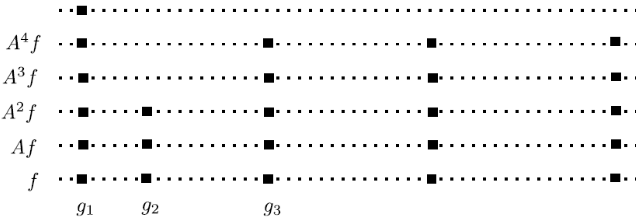
$$\langle A^n f, g \rangle \text{ for } 0 \leq n < L(g), \quad g \in \mathcal{G} \tag{1}$$

where $\mathcal{G}$ is a countable set of vectors in $\mathcal{H}$ and $L : \mathcal{G} \mapsto \mathbb{N} \cup \{\infty\}$ a function.

Main problem:

Recover the vector $f \in H$ from measurements (1)

Special case: $A_i = A$ in a separable Hilbert space II



In particular we are interested in stable recovery when for every $f \in \mathcal{H}$

$$C_1 \|f\|^2 \le \sum_{g \in \mathcal{G}} \sum_{0 \le n < L(g)} |\langle A^n f, g \rangle|^2 \le C_2 \|f\|^2$$

where $C_1, C_2 > 0$ are absolute constants.

Special case: $A_i = A$ in a separable Hilbert space III

Using the fact that

$$\langle Af, g\rangle = \langle f, A^* g\rangle$$

we get the following equivalent formulation.

Proposition

1. *Any $f \in \mathcal{H}$ can be recovered from $\{\langle A^n f, g\rangle\}_{g\in\mathcal{G}, 0\le n<L(g)}$ if and only if the system*

$$\{(A^*)^n g\}_{g\in\mathcal{G}, 0\le n<L(g)}$$

   is complete in $\mathcal{H}$.

2. *Any $f \in \mathcal{H}$ can be recovered from $\{\langle A^n f, g_i\rangle\}_{g\in\mathcal{G}, 0\le n<L(g)}$ in a stable way if and only if the system*

$$\{(A^*)^n g\}_{g\in\mathcal{G}, 0\le n<L(g)}$$

   is a frame in $\mathcal{H}$.

Special case: $A_i = A$ in a separable Hilbert space IV

Theorem

If $\{A^n g\}_{g\in\mathcal{G},\, n\geq 0}$ is a frame in $\mathcal{H}$ then for every $f \in \mathcal{H}$,

$$(A^*)^n f \to 0 \text{ as } n \to \infty. \quad (2)$$

Corollary

For any unitary operator $A : \mathcal{H} \to \mathcal{H}$ and any set of vectors $\mathcal{G} \subset \mathcal{H}$, $\{A^n g\}_{g\in\mathcal{G},\, n\geq 0}$ is not a frame in $\mathcal{H}$.

We don't know if (2) is a sufficient condition for the existence of a frame by iterations.

Theorem

If A is a contraction (i.e. $\|A\| \leq 1$), and (2) holds, then we can choose $\mathcal{G} \subseteq \mathcal{H}$ such that $\{A^n g\}_{g\in\mathcal{G},\, n\geq 0}$ is a Parseval frame.

Frames = generalization of orthonormal bases

Frame definition

A sequence $F = \{\mathbf{f}_i\}_{i\in I}$ (I-count. index set) of $\mathcal{H} \setminus \{0\}$ is a *frame* for $\mathcal{H}$, if there exist $0 < C \leq D < \infty$ such that

$$C\|\mathbf{f}\|^2 \leq \sum_{i\in I} |\langle \mathbf{f}, \mathbf{f}_i\rangle|^2 \leq D\|\mathbf{f}\|^2 \text{ for all } \mathbf{f} \in \mathcal{H}. \tag{4}$$

- The *frame operator* $S\mathbf{f} := \sum_{i\in I}\langle \mathbf{f}, \mathbf{f}_i\rangle \mathbf{f}_i$ is invertible
- For each frame F of $\mathcal{H}$ there exists at least one *dual* frame $G = \{\mathbf{g}_i\}_{i\in I}$, satisfying

$$\mathbf{f} = \sum_{i\in I}\langle \mathbf{f}, \mathbf{f}_i\rangle \mathbf{g}_i = \sum_{i\in I}\langle \mathbf{f}, \mathbf{g}_i\rangle \mathbf{f}_i \text{ for all } \mathbf{f} \in \mathcal{H}. \tag{5}$$

- The set $\{\mathbf{g}_i = S^{-1}\mathbf{f}_i\}_{i\in I}$ is called the *canonical dual* frame.
- The frame F is *C-tight*, if $C = D$ in (4), and

$$\mathbf{f} = \frac{1}{C}\sum_{i\in I}\langle \mathbf{f}, \mathbf{f}_i\rangle \mathbf{f}_i = \frac{1}{C}S\mathbf{f} \text{ for all } \mathbf{f} \in \mathcal{H}. \tag{6}$$

Consider $\mathcal{G} = \{\mathbf{f}_i\}_{i \in I} \subseteq \mathcal{H}$, and a bounded operator $A : \mathcal{H} \to \mathcal{H}$.

$$\text{If } \; F_{\mathcal{G}}^{\mathbf{L}}(A) := \cup_{i \in I}\{A^j \mathbf{f}_i | \mathbf{f} \in \mathcal{G}\}_{j=0}^{L_i} \text{ is a frame for } \mathcal{H}, \tag{7}$$

then we call (7) a *dynamical frame*, gen. by A and $\mathbf{f}_i$.

Theorem 1

Let $F_{\mathcal{G}}^{\mathbf{L}}(A)$ be a dynamical frame for $\mathcal{H}$, with frame operator S. The canonical dual frame of $F_{\mathcal{G}}^{\mathbf{L}}(A)$ is also dynamical, generated by $B := S^{-1}AS$ and $\mathbf{g}_i = (S^{-1}\mathbf{f}_i)$ i.e.

$$\mathbf{f} = \sum_{i \in I} \sum_{j=0}^{L_i} \langle \mathbf{f}, A^j \mathbf{f}_i \rangle B^j \mathbf{g}_i \quad \text{for every } \mathbf{f} \in \mathcal{H}. \tag{8}$$

Dynamical Frames and Scalability

(joint work with Y. Kim, arxiv: 1608.05622v1)

If the frame F is tight then $S^{-1} \equiv I$. In general, computing S^{-1} can be a challenging problem.
If there exist scaling coefficients $w_i \geq 0$, $i \in I$, such that $F_w := \{w_i \mathbf{f}_i\}_{i \in I}$ is a tight frame, then we call F a *scalable* frame.

Property

Say $F_{\mathcal{G}}^{\mathbf{L}}(A) = \cup_{i \in I} \{A^j \mathbf{f}_i | \mathbf{f} \in \mathcal{G}\}_{j=0}^{L_i}$ is a scalable frame; then

$$\mathbf{f} = \sum_{i \in I} \sum_{j=0}^{L_i} w_{ij}^2 \langle \mathbf{f}, A^j \mathbf{f}_i \rangle A^j \mathbf{f}_i \quad \text{for every } \mathbf{f} \in \mathcal{H}. \tag{9}$$

Example: $\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $A = \begin{pmatrix} \cos(2\pi/3) & -\sin(2\pi/3) \\ \sin(2\pi/3) & \cos(2\pi/3) \end{pmatrix}$.
$\{\mathbf{e}_1, A\mathbf{e}_1, A^2\mathbf{e}_1\}$ is a scalable frame for $\mathbb{R}^2$ (mercedes).

BALL STATE UNIVERSITY

Let $\mathcal{H} = \mathbb{R}^n$, and let $\{\mathbf{e}_j\}_{j=1}^n$ be the std orthonormal basis of $\mathcal{H}$.

Theorem *

Let $\mathcal{G} = \{\mathbf{e}_{k_1}, \ldots, \mathbf{e}_{k_p}\}$, $p < n$, and $\mathbf{L} = (L_1, \ldots, L_p) \in \mathbb{Z}_+^p$.
Let A be an operator on $\mathcal{H}$.
If B is a unitary operator on $\mathcal{H}$, then TFAE

- $\cup_{s=1}^p \{A^j \mathbf{e}_{k_s}\}_{j=0}^{L_s}$ is a (scalable) frame
- $\cup_{s=1}^p \{C^j \mathbf{g}_s\}_{j=0}^{L_s}$ is a (scalable) frame,

where $C := B^{-1}AB$, and $\mathbf{g}_s := B^{-1}\mathbf{e}_{k_s}$, $s = 1, \ldots, p$.

Note: if $A = URU^T$ is a Schur decomposition of A, with a unitary matrix U and a matrix of Schur form R, then TFAE

(i) $\cup_{s=1}^p \{A^j \mathbf{e}_{k_s}\}_{j=0}^{L_s}$ is a (scalable) frame for $\mathcal{H}$,

(ii) $\cup_{s=1}^p \{R^j \mathbf{v}_s\}_{j=0}^{L_s}$ is a (scalable) frame for $\mathcal{H}$.

Scalable Dynamical Frames: Normal Operators in $\mathbb{R}^n$ (I)

Let $A = UDU^T$ be a normal operator on $\mathcal{H}$, $D = diag(a_1, \dots, a_n)$, U-orthogonal;
let $\mathbf{e}_{l_k}$, $k = 1, \dots, s$, $s \leq n$, be standard basis vectors for $\mathcal{H}$.
TFAE:

- $F_{\mathcal{G}}(A) = \cup_{s=1}^{p} \{A^j \mathbf{e}_{l_s} \,|\, j = 0, 1, \dots, L_s\}$ is a scalable frame
- $\cup_{s=1}^{p} \{D^j \mathbf{v}_s \,|\, j = 0, 1, \dots, L_s\}$ is a scalable frame for $\mathbb{R}^n$

where $\mathbf{v}_s = U^T \mathbf{e}_{l_s} = (x_s(1), \dots, x_s(n))^T$, $1 \leq s \leq p$.

The scaling coefficients $w_{s,t}$ of $F_{\mathcal{G}}(A)$ are solutions to (*):

$$\sum_{s=1}^{p} \|x_s(i)\|^2 \left[w_{s,0}^2 + w_{s,1}^2 \|a_i\|^2 + \cdots + w_{s,L_s}^2 \|a_n\|^{2L_s} \right] = 1,$$

$$\sum_{s=1}^{p} x_s(i) \bar{x_s}(j) \left[w_{s,0}^2 + w_{s,1}^2 a_i \bar{a}_j + \cdots + w_{s,L_s}^2 (a_i \bar{a}_j)^{L_s} \right] = 0,$$

for all $i, j = 1, \dots, n$, $i \neq j$.

Example 1: Let

$$D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad ; \mathbf{v}_1 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \ \mathbf{v}_2 = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}.$$

The frame $\{\mathbf{v}_1, D\mathbf{v}_1, \mathbf{v}_2, D\mathbf{v}_2\}$ is a scalable frame in $\mathbb{R}^3$, with weights $w_{k,i} = 0.5$, $0 \leq i \leq 1$, $1 \leq k \leq 2$.

Example 2: Let $\mathbf{v}_1 = (x_1, x_2, x_3)^T$, $\mathbf{v}_2 = (y_1, y_2, y_3)^T$ and $D = diag(a, b, 0)$, where $ab \neq 0$, $1 + ab < 0$. Set

$$x_1 = \pm \frac{1}{a\sqrt{b^2(a^2+1) - a^2(1+ab)}}, \ x_2 = \pm \frac{1}{b\sqrt{a^2(b^2+1) - b^2(1+ab)}},$$

$$x_3 = \pm \frac{\sqrt{-(1+ab)}}{ab}, \text{ and } \ y_3 = \pm\sqrt{1 - x_3^2}, \ y_1 = -\frac{x_1 x_3}{y_3}, \ y_2 = -\frac{x_2 x_3}{y_3}.$$

Then $\{\mathbf{v}_1, A\mathbf{v}_1, \mathbf{v}_2, A\mathbf{v}_2, A^2\mathbf{v}_2\}$ is a tight frame of $\mathbb{R}^3$.

Theorem 2

Let $D = diag(a_1, \cdots, a_n)$, where $a_1, \ldots, a_n \in \mathbb{R}$, and let $\mathbf{v}_s = (x_s(1), \ldots, x_s(n))^T \in \mathcal{H}$, $s \in \{1, \cdots, p\}$, $p \geq 1$.

The sequence $\cup_{s=1}^{p} \{D^j \mathbf{v}_s \mid j = 0, 1, \ldots, L_s\}$ is a scalable frame for $\mathcal{H}$ if and only if there exist scaling coefficients $w_{s,0}, w_{s,1}, \ldots, w_{s,L_s}$, $s = 1, \ldots, p$, which satisfy conditions (*).

- $\{D^j \mathbf{v}\}_{j=0}^{L}$ is never a scalable frame for $\mathbb{R}^n$ (we need $p \geq 2$)

By Theorem* and Theorem 2, the following result holds true:

Theorem 3

Let A be a normal operator for $\mathcal{H}$ and $A = UDU^T$ be the orthogonal diagonalization of A. Let $\mathbf{e}_{l_k}$, $1 \leq k \leq s$ be standard basis vectors for some $s \leq n$.

The sequence $\cup_{k=1}^{s}\{A^j \mathbf{e}_{l_k} \mid 0 \leq j \leq L_s\}$ is a scalable frame of $\mathcal{H}$ if an only if there there exist scaling coefficients $w_{k,0}, w_{k,1}, \dots, w_{k,L_k}$, $1 \leq k \leq s$, which satisfy conditions (*).

Example

Let $a, b, c, d \in R$, $a > 0$ and $abcd \neq 0$. Let

$$A = \begin{pmatrix} a & c \\ b & d \end{pmatrix} \in \mathbb{R}^{2\times 2}, \quad \mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \in \mathbb{R}^2.$$

$\{\mathbf{e}_1, A\mathbf{e}_1, A^2\mathbf{e}_1\}$ is a scalable frame for $\mathbb{R}^2$ iff $0 < -\frac{ac}{bd} < 1$.

Let $A = \begin{pmatrix} a & c & 0 & 0 \\ b & d & 0 & 0 \\ 0 & 0 & a & c \\ 0 & 0 & b & d \end{pmatrix}, \mathbf{f}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \mathbf{f}_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \ 0 < -\dfrac{ac}{bd} < 1.$

Then $\{\mathbf{f}_1, A\mathbf{f}_1, A^2\mathbf{f}_1, \mathbf{f}_2, A\mathbf{f}_2, A^2\mathbf{f}_2\}$ is a scalable frame for $\mathbb{R}^4$.

Theorem 4

Let F_i be a (scalable) frame for $\mathbb{R}^{n_i}$, $i = 1, \dots p$. Let

$$G := \begin{pmatrix} F_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & F_p \end{pmatrix}. \quad (10)$$

Then the system G is a (scalable) frame for $\mathbb{R}^N$, $N = n_1 + \dots + n_p$.

Let $A_s \in \mathbb{R}^{n_s \times n_s}$, $1 \leq s \leq p$, $\sum_{s=1}^{p} n_s = N$, and let $A \in \mathbb{R}^{N \times N}$ be a block-diagonal matrix with $A_1, \dots, A_p$ on its diagonal.

Let $\mathbf{v} \in \mathbb{R}^{n_s}$. $\mathbf{v} \in \mathbb{R}^{n_s}$ is *well-embeded* in $\mathbf{f} \in \mathbb{R}^N$ w.r.t. A, if $\mathbf{f}(j) = \mathbf{v}(i)$, when $j = n_1 + \dots n_s + i$, and $\mathbf{f}(j) = 0$, otherwise.

Theorem 5

Let $A_s \in \mathbb{R}^{n_s \times n_s}$, $1 \leq s \leq p$, where $n_1 + \cdots + n_p = N$. Let $A \in \mathcal{H}^{N \times N}$ be a block-diagonal matrix constructed by distributing matrices $A_1, \ldots, A_p$ along its diagonal.
Let $\mathbf{f}_{s,1}, \ldots, \mathbf{f}_{s,m_s} \in \mathbb{R}^N$, $1 \leq s \leq p$ be the m_s well-embedded vectors $\mathbf{v}_{s,1} \ldots, \mathbf{v}_{s,m_s} \in \mathbb{R}^{n_s}$, $1 \leq s \leq p$.

TFAE

- $\{A_s^j \mathbf{v}_{s,k} \mid 1 \leq k \leq m_s\}_{j=0}^{L_{s,k}}$ is a (scalable) frame of $\mathbb{R}^{n_s}$, $1 \leq s \leq p$.
- $\cup_{s=1}^{p} \{A^j \mathbf{f}_{s,k} \mid 1 \leq k \leq m_s\}_{j=0}^{L_{s,k}}$ is a (scalable) frame of $\mathbb{R}^N$.

References

[1] R. Aceska, A. Aldroubi, J. Davis, and A. Petrosyan, Dynamical sampling in shift invariant spaces, *Contemp. Math. Of the AMS*, 603:139-148, 2013.

[2] R. Aceska, and S. Tang, Dynamical Sampling in Hybrid Shift-invariant Spaces, *Contemp. Math. Of the AMS*, 626: 149-166, 2014.

[3] R. Aceska, A. Petrosyan, and S. Tang, Multidimensional Signal Recovery in Discrete Evolution Systems via Spatiotemporal Trade Off, *Sampling Theory in Signal and Image Processing*, 14(2): 153-169, 2015.

[4] R. Aceska, A. Petrosyan, and S. Tang, Dynamical sampling of two-dimensional temporally-varying signals, *Proc. 2015 Int. Conf. Sampling Theory and Applications (SampTA)*, 440-443, IEEE, 2015.

[5] A. Aldroubi, J. Davis and I. Krishtal, Dynamical Sampling: Time Space Trade-off, *Appl. Comput. Harmon. Anal.*, **34**(3), 495–503, 2013.

[6] A. Aldroubi, J. Davis and I. Krishtal, Exact Reconstruction of Signals in Evolutionary Systems Via Spatiotemporal Trade-off, *J. Fourier Anal. Appl.*, **21**(1), 11–31, 2015.

[7] A. Aldroubi, C. Cabrelli, U. Molter, and Sui Tang, Dynamical sampling, Submitted. Available at http://arxiv.org/abs/1409.8333.

[8] A. Aldroubi, and A. Petrosyan, Dynamical sampling and systems from iterative actions of operators, Submitted. Available at http://arxiv.org/abs/1606.03136.

[9] A. Aldroubi, J. Davis and I. Krishtal, Dynamical Sampling: Time Space Trade-off, *Appl. Comput. Harmon. Anal.*, **34**(3), 495–503, 2013.

[10] A. Aldroubi, J. Davis and I. Krishtal, Exact Reconstruction of Signals in Evolutionary Systems Via Spatiotemporal Trade-off, *J. Fourier Anal. Appl.*, **21**(1), 11–31, 2015.

[11] A. Aldroubi, C. Cabrelli, A. F. Cakmak, U. Molter, and A. Petrosyan, Iterative actions of normal operators, Submitted. Available at http://arxiv.org/abs/1602.04527.

BALL STATE UNIVERSITY.