MWAA 2019
Purdue University Fort Wayne
October 4–6th

Friday		
15:30–16:00	**Coffee**	
16:00–17:00	**S. Bell**	The theorem that is too good to be true and how it made me very happy
Saturday		
09:00–09:40	**Posters & Coffee**	
09:40–10:20	**J. Anderson**	Adaptations of an integral form of the maximum principle for nonlinear diffusion equations
10:30–11:10	**S. Dai**	On existence of entire solutions of the Poincaré-Lelong equations in $\mathbb{C}^n$
11:20–12:00	**A. Grenschaw**	Parabolic measure
12:00–14:00	**Lunch**	
14:00–14:40	**T. Anderson**	Spheres, primes, and triangles: tales from the interface of harmonic analysis and number theory
14:50–15:30	**M. Carnovale**	The Falconer distance conjecture and Roth's theorem
15:30–16:00	**Coffee**	
16:00–16:40	**J. Baik**	Totally asymmetric simple exclusion process
16:50–17:30	**D. Kemp**	Decoupling for real analytic surfaces exhibiting zero curvature
Sunday		
09:00–10:00	**Posters & Coffee**	
10:00–10:40	**S. Hussung**	Nontraditional notions of polynomial ordering with computational applications
10:50–11:30	**J. Criado del Rey**	An equilibrium problem on the sphere with two equal charges
11:40–12:20	**S. Borodachov**	Optimal recovery of Sobolev classes using multivariate splines

Plenary Talks

Jeff Anderson
Purdue University Fort Wayne

Adaptations of an integral form of the maximum principle for nonlinear diffusion equations

A widely used estimate for solutions of diffusion equations is established using a Sobolev-like embedding and a recursive inequality. Of interest on its own, the development and resulting upper bound applies in very general settings. However, this method typically stays in the background, providing a foundation on which further results may be built. Recently, we have applied such an approach (both the method and unfortunately relegating it to the background) toward the study of global solvability for nonlinear diffusion equations with boundary flux driven by memory interactions. We discuss the set up of the integral maximum principle in this context, a resulting local in time maximum estimate, and further adaptation that yields global solvability for the case of degenerate diffusion. In the remaining so-called fast diffusion case, still further modification is needed. On the other hand, the global solvability result for these models turns out to be sharp.

Tess Anderson
Purdue University West Lafayette

Spheres, primes, and triangles: tales from the interface of harmonic analysis and number theory

Pioneered by Bourgain, the fusion of Fourier analytic and number theoretic techniques in novel ways have led to a variety of discrete operator bounds where continuous techniques fail. Moreover, many distributional questions can be answered in a quantitatively strong way by knowing such bounds. We discuss recent work pertaining to distribution of primes on spheres, higher degree spherical maximal functions and three-point configurations.

Jinho Baik
University of Michigan

Totally asymmetric simple exclusion process

In a traffic flow of cars in a single lane, a slowly moving car affects the following vehicles. The totally asymmetric simple exclusion (TASEP) process is a simple probabilistic model for such a system. It is one of the fundamental models in the interacting particle systems. Furthermore, it is one of the first models for which the law of the fluctuations of the particle locations from their expected locations after a long time is determined. One way to obtain the probability distribution of a particle location is by solving the Kolmogorov forward equation, which is a non-constant coefficient linear PDE, and then taking the large time limit of the Fredholm determinant formula that arises. We discuss how this procedure works for the TASEP on the line and also for the TASEP on a ring which was studied recently.

Steven Bell
Purdue University West Lafayette

The theorem that is too good to be true and how it made me very happy

Joint work with Björn Gustafsson

Mergelyan's theorem about approximating continuous functions on compact sets by complex rational functions has often been called a theorem that is too good to be true — but it is! I will describe my 40 year quest to better understand the relationship between the Bergman and Szegő kernels in planar domains and how Mergelyan's theorem helped me check off this thing near the top of my bucket-list.

Sergiy Borodachov
Towson University

Optimal recovery of Sobolev classes using multivariate splines

We will start by discussing the general setting of the problem of optimal global recovery of functions from given discrete data. Then we will consider this problem on different classes of multivariate functions and show how multivariate splines arise as exact solutions. If time permits, certain asymptotic results will also be discussed as well as the relation of optimal recovery to certain discrete geometric problems.

Marc Carnovale
Ohio State University

The Falconer distance conjecture and Roth's theorem

Roth's theorem states that a subset of $\{1, ..., N\}$ containing more than $cN/\log\log N$ elements must contain non-trivial three-term arithmetic progressions. The Falconer distance conjecture asks whether every subset $E \subset \mathbb{R}^d$, $d \geqslant 2$, with Hausdorff dimension greater than $d/2$ must have a distance set of positive Lebesgue measure. Combining non-trivial techniques from the study of both, joint with Steven Senger we obtain a result on the largeness of certain point configurations within supports of measures whose Fourier transforms lie in L^q with small enough norm. We motivate the question with a discussion of maximal operators, restriction theorems, and evidence of a deeper connection to arithmetic-combinatorial underpinnings.

Juan Criado del Rey
KU Leuven

An equilibrium problem on the sphere with two equal charges

Joint work with Alan Groot and Arno Kuijlaars

We study the weighted equilibrium measure associated to a logarithmic external field generated by two point charges on the two-dimensional sphere. When the charges are small, the droplet is known to be the complement of two spherical caps, but as soon as the charges become large, the shape of the droplet changes. In this talk we will see how can we describe the shape of the droplet and what is the role of non-standard orthogonal polynomials in the solution of the problem.

Shaoyu Dai
Jinling Institute of Technology — visiting Purdue Fort Wayne

On existence of entire solutions of the Poincaré-Lelong equations in $\mathbb{C}^n$

Joint work with Yifei Pan

Lelong first studied the Poincaré-Lelong equation $\sqrt{-1}\partial\bar{\partial}u = f$, where f is a d-closed $(1,1)$-form defined on $\mathbb{C}^n$, by reducing it to Poisson's equation $\Delta u = \text{trace}(f)$, assuming suitable growth conditions on f. We prove the existence of entire solutions of the Poincaré-Lelong equations for any f that is in the weighted Hilbert space with Gaussian measure, i.e., $L^2_{(1,1)}(\mathbb{C}^n, e^{-|z|^2})$. One of the key ideas is to prove a L^2 version of the Poincaré Lemma for 2-forms, and apply Hörmander's L^2 solutions for Cauchy-Riemann equations.

Alyssa Genschaw
University of Connecticut

Parabolic measure

We will discuss parabolic measure associated to a uniformly parabolic divergence form operator. We will give a brief overview of some recent results, including a Bourgain-type estimate, a criterion for non-doubling parabolic measure to satisfy a weak reverse Hölder inequality, and that BMO-solvability implies scale invariant quantitative absolute continuity of parabolic measure with respect to surface measure.

Steven Hussung
Indiana University

Nontraditional notions of polynomial ordering with computational applications

Nontraditional notions of polynomial degree and ordering will be discussed, with generalizations of several potential-theoretic results proceeding from Siciak-Zaharjuta type theory. I will focus particularly on numerical aspects, and will present generalizations of Fekete and Leja points in this setting, as well as discrete versions of each. I will also present two numerical asymptotic descriptions of the potential theoretic extremal function, generalized to this setting.

Dominique Kemp
Indiana University

Decoupling for real analytic surfaces exhibiting zero curvature

The celebrated l^2 decoupling theorem of Jean Bourgain and Ciprian Demeter presented a new perspective on a range of problems related to hypersurfaces with nonzero Gaussian curvature, such as exponential sum estimates, additive energy estimates, local smoothing, and counting solutions to Diophantine inequalities. The same authors also extended their theory to the n-dimensional cone. Following their steps, we prove optimal l^2 decoupling results for the remaining class of zero-curvature two-dimensional surfaces (the so-called tangent surfaces). We are also able to prove a decoupling theorem for the real analytic surfaces of revolution. These results should be viewed as partial progress toward the goal of proving a decoupling theorem for arbitrary real analytic hypersurfaces.

Poster Presentations

Hanan Aljubran

On asymptotic expansion of the expected number of real zeros of random polynomials spanned by OPUC

Joint work with Maxim Yattselev

Let $\{\varphi_i\}_{i=0}^{\infty}$ be a sequence of orthonormal polynomials on the unit circle with respect to a positive Borel measure μ that is symmetric with respect to conjugation. Asymptotic behavior of the expected number of real zeros, say $\mathbb{E}_n(\mu)$, of random polynomials

$$P_n(z) := \sum_{i=0}^{n} \eta_i \varphi_i(z),$$

where $\eta_0, \ldots, \eta_n$ are i.i.d. standard Gaussian random variables is studied. When μ is the arclength measure such polynomials are called Kac polynomials and it was shown by Wilkins that $\mathbb{E}_n(|d\xi|)$ admits an asymptotic expansion of the form

$$\mathbb{E}_n(|d\xi|) \sim \frac{2}{\pi}\log(n+1) + \sum_{p=0}^{\infty} A_p (n+1)^{-p}$$

(Kac himself obtained the leading term of this expansion). We generalize the result of Wilkins to the case where μ is absolutely continuous with respect to arclength measure and its Radon-Nikodym derivative extends to a holomorphic non-vanishing function in some neighborhood of the unit circle. In this case $\mathbb{E}_n(\mu)$ admits an analogous expansion with coefficients the A_p depending on the measure μ for $p \geqslant 1$ (the leading order term and A_0 remain the same).

Adam Coffman

Complex linear algebra without complex numbers

Motivated by calculations in complex vector bundles, this expository poster shows how a complex structure operator J, satisfying $J \circ J = -I$ on a real vector space V, can be used to define complex linear maps, complex tensor products, and a complex trace, in terms of only the real linear structure.

Evangelos Nastas

Some proofs on an inequality related to a theorem by Malliavin

This work is devoted to an inequality that's an inversion formula related to the Cauchy integral of a distribution function. It yields a theorem proven by Malliavin. Alternatives to the inequality are presented.

Menuja Perera

A transformation rule associated to $P-$ extremal functions and holomorphic mappings

Joint work with Norm Levenberg

M. Klimek has proved a transformation rule for standard extremal functions V_K subject to holomorphic mappings f, under several other conditions on f. Here, by considering the $P-$ extremal functions $V_{P,K}$ for two different convex bodies P, we present a similar result that we have discovered in the setting of $P-$ pluripotential theory. Three types of candidates

that satisfy the hypotheses will also be illustrated. This result is particularly applicable in determining $V_{P,K}$ for some non-product sets P.

Ali Pirhadi

Real zeros of random cosine polynomials with pairwise equal blocks of coefficients

It is well known that the expected number of real zeros of a random cosine polynomial

$$V_n(x) = \sum_{j=0}^{n} a_j \cos(jx),\ x \in (0, 2\pi)$$

with the a_j being standard Gaussian i.i.d. random variables is asymptotically $2n/\sqrt{3}$. On the other hand, some of the previous works on the random cosine polynomials with dependent coefficients show that such polynomials have at least $2n/\sqrt{3}$ expected real zeros lying in one period. We investigated three classes of random cosine polynomials with pairwise equal blocks of coefficients. First, we see that a random cosine polynomial with the blocks of coefficients being of a fixed length ℓ and satisfying $A_{2j} = A_{2j+1}$ possesses the same expected real zeros as the classical case. Afterwards, we observe that the case containing only two equal blocks of coefficients has significantly more real zeros compared with those of the classical case. Finally, we see that the asymptotics of the expected number of real roots of a random cosine polynomial with *palindromic* blocks is $K_\ell \cdot 2n/\sqrt{3}$, where the constant K_ℓ (depending only on ℓ) is greater than 1, and can be explicitly represented by a double integral formula. That is to say, such polynomials have slightly more expected real zeros compared with the classical case with i.i.d. coefficients.

Cong Zhou

Hinčin's theorem for additive free convolutions of nontracial R-diagonal $*$-distributions

Hinčin proved that any limit law associated with a triangular array of uniformly infinitesimal random variables is infinitely divisible. Subsequently, an analogous result for additive free convolutions of tracial R-diagonal $*$-distributions was proved by the author. We prove an analogous result for additive free convolutions of nontracial R-diagonal $*$-distributions.