# MWAA 2023

## Indiana University - Purdue University Indianapolis

## October 13–15th

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|---|---|---|
| **Friday** | | |
| 15:30–16:00 | **Coffee** | |
| 16:00–17:00 | **B. Wick** | Two weight inequalities for Calderón-Zygmund operators |
| **Saturday** | | |
| 09:00–09:40 | **Posters & Coffee** | |
| 09:40–10:20 | **R. Gharakhloo** | Strong Szegő limit theorems for bordered and framed Toeplitz determinants |
| 10:30–11:10 | **L. Vivas** | Wiegerinck conjecture on Bergman Spaces |
| 11:20–12:00 | **R. Buckingham** | Asymptotics of rational Painlevé V functions |
| 12:00–14:00 | **Lunch** | |
| 14:00–14:40 | **E. Pozzi** | Composition operators on general complex domains |
| 14:50–15:30 | **V. Bailey** | Frames via unilateral iterations of bounded operators |
| 15:30–16:00 | **Coffee** | |
| 16:00–16:40 | **S. Nivoche** | Behavior of multipoled pluricomplex Green's functions in connection with algebraic geometry problems |
| 16:50–17:30 | **S. Wu** | The $SL_2$ Kakeya problem |
| 18:30–21:00 | **Conference Dinner** | **The Rathskeller Restaurant** |
| **Sunday** | | |
| 09:00–09:40 | **Posters & Coffee** | |
| 09:40–10:20 | **T. Sorokina** | Intrinsic supersmoothness of piecewise multivariate functions |
| 10:30–11:10 | **L. Bos** | Optimal prediction measures |
| 11:20–12:00 | **E. Blackstone** | Large gap asymptotics for the Bessel kernel determinant |

# Plenary Talks

**Victor Bailey**
**University of Oklahoma**

## Frames via Unilateral Iterations of Bounded Operators

*Joint work with Carlos Cabrelli*

Dynamical Sampling is, in a sense, a hypernym classifying the set of inverse problems arising from considering samples of a signal and its future states under the action of a bounded linear operator. Recent works in this area consider questions such as when can a given frame for a separable Hilbert space, $\{f_k\}_{k\in I} \subset H$, be represented by iterations of an operator on a single vector and what are necessary and sufficient conditions for a system, $\{T^n\varphi\}_{n=0}^{\infty} \subset H$, to be a frame? In this talk, we will discuss the connection between frames given by iterations of a bounded operator and the theory of model spaces in the Hardy-Hilbert space as well as necessary and sufficient conditions for a system generated by the orbit of a pair of commuting bounded operators to be a frame.

**Elliot Blackstone**
**University of Michigan**

## Large Gap Asymptotics for the Bessel Kernel Determinant

We give a brief introduction to Riemann–Hilbert problems (RHPs) and describe the ideas of the Deift–Zhou steepest descent method applied to a particular $2 \times 2$ matrix RHP. This particular RHP appears when one studies large gap asymptotics for the Bessel kernel determinant.

**Len Bos**
**Università di Verona**

## Optimal Prediction Measures

Suppose that $K \subset \mathbb{C}^d$ is a compact set. For data given on $K$ (with random errors) it is possible, by means of polynomial regression, to *predict* (or extrapolate) a value at a point $z_0$ *exterior* to $K$. An optimal prediction measure is the probability measure on $K$ which describes the data distribution on $K$ for which the predicted value has least variance. We will discuss this problem and its relation to another classical approximation problem, give some examples, and discuss some conjectures and open problems.

**Robert Buckingham**
**University of Cincinnati**

## Asymptotics of Rational Painlevé V Functions

*Joint work with Matthew Satter of the University of Cincinnati*

The Painlevé functions are a family of ordinary differential equations with myriad applications to mathematical physics and probability. The rational solutions of these equations have drawn attention for the remarkable geometric structure of their zeros and poles. We study the family of rational solutions of the Painlevé-V equation built from the so-called Umemura polynomials. We derive a new Riemann-Hilbert representation and use it to obtain the boundary of the pole region and the large-degree behavior in the pole-free region.

**Roozbeh Gharakhloo**
**University of California, Santa Cruz**

### Strong Szegő Limit Theorems for Bordered and Framed Toeplitz Determinants

Structured determinants and their large size asymptotics are at the core of important questions in various areas, including random matrix theory and statistical mechanics. In this talk we describe the general framework for obtaining Strong Szegő Limit Theorems for (multi-)bordered, semi-framed, and (multi-)framed Toeplitz determinants. These determinants appear in the study of ensembles of nonintersecting paths, entanglement entropy for disjoint subsystems in the XX spin chain, and off-diagonal correlations in the two-dimensional Ising model.

**Stéphanie Nivoche**
**Université Côte d'Azur**

### Behavior of Multipoled Pluricomplex Green's Functions in Connection with Algebraic Geometry Problems

The Nagata Conjecture governs the minimal degree required for a plane algebraic curve to pass through a collection of $r$ general points in the projective plane $\mathbb{P}^2$ with prescribed multiplicities. The "SHGH" Conjecture governs the dimension of the linear space of these polynomials. We formulate transcendental versions of these conjectures in term of pluripotential theory and we are making some progress.

**Elodie Pozzi**
**Saint Louis University**

### Composition Operators on General Complex Domains

*Joint work with B.R Choe, H. Koo, and W. Smith*

The composition operator is a well-known linear operator acting boundedly on several analytic function spaces. There is a profusion of remarkable results on the composition operator on the Hardy space $H^p$ and the Bergman space $A^p$ of the unit disk $\mathbb{D}$, for instance. Less is known on domains with less regular boundary. In this talk, we give an overview of some results on less smooth domains. We will give a new boundedness result on Carleson domains, known as domains $\Omega$ (bounded or unbounded) such that for any Carleson measure $\mu$, the $L^1(\Omega, d\mu)$-norm of a function in $H^1$ is less than $C(\mu)\|f\|_{L^1(\partial\Omega)}$. We will provide some simple general domains for which the composition operator is bounded.

**Tatyana Sorokina**
**Towson University**

### Intrinsic Supersmoothness of Piecewise Multivariate Functions

Multivariate polynomial splines represent the simplest example of piecewise functions. They are well-known tools in computer-aided design and manufacturing, image processing and numerical PDEs. They have been studied extensively in approximation theory and numerical analysis. Since splines are piecewise polynomials of fixed degree and smoothness defined over suitable partitions of domains in $\mathbb{R}^n$, they form vector spaces. Spline analysis usually includes determining the dimension of such spaces, finding bases that can be used to solve interpolation problems, and computing approximation order. In this talk we will first address a lesser-known aspect of multivariate splines: intrinsic

supersmoothness. In the definition of a spline, there is a prescribed value for the global smoothness of the piecewise function. Intrinsic supersmoothness is a phenomenon that describes additional non-prescribed smoothness across certain faces that appears in almost all multivariate splines except in the univariate setting. We show that the geometry of the underlying partition determines the associated supersmoothness. We demonstrate how intrinsic supersmoothness can be used to compute dimensions of splines. We next show how the concept of intrinsic supersmoothness generalizes to non-polynomial piecewise functions.

**Liz Vivas**
**The Ohio State University**

### Wiegerinck Conjecture on Bergman Spaces

*Joint work with A-K. Gallagher and P. Gupta*

Wiegerinck proved that the Bergman space over any domain in the complex plane is either trivial or infinite dimensional. In this talk I will discuss various generalizations and open questions related to this theorem. I will survey the case of the complex plane being replaced by $\mathbb{C}^n$ as well as a domain in a compact Riemann Surface.

**Brett Wick**
**Washington University in St. Louis**

### Two Weight Inequalities for Calderón-Zygmund Operators

Calderón-Zygmund operators, which arise naturally in partial differential equations and complex analysis, are integral operators that are associated to a kernel possessing a singularity on the diagonal. Understanding the mapping properties of these operators on various function spaces is an important area of current research. There is a proof strategy that exists, "T1 Theorems", that utilizes natural necessary testing conditions to provide sufficient conditions to verify the boundedness of the Calderón-Zygmund operator. Additional tools in the proof are the use of dyadic harmonic analysis techniques. In this talk we will discuss some recent results about the behavior of Calderón-Zygmund operators on weighted spaces in various settings and outline some of the main ideas behind the proof and provide some motivation to consider such questions.

**Shukun Wu**
**Indiana University**

### The $SL_2$ Kakeya Problem

*Joint work with Nets Katz and Joshua Zahl*

The $SL_2$ Kakeya problem is a special case of the three-dimensional Kakeya conjecture, and it is surprisingly connected to the restricted projection problem in geometric measure theory. In this talk, I will first survey the Kakeya conjecture. Then I will discuss a solution to the $SL_2$ Kakeya problem and its connection to the restricted projection theorem.

# Poster Presentations

**Victor Júlio Alves de Souza**

### The Rational Weighted Pólya-Tchebotarev Problem

The Pólya-Tchebotarev problem is a classical problem in geometric function theory given by the maximization of the logarithmic energy over a family of compact sets on the complex plane that contains a fixed finite set. We formulate the rational weighted Pólya-Tchebotarev problem, in which the maximization is given with the presence of a rational external field, and comment its connection to asymptotic of orthogonal polynomials through the Gonchar-Rakhmanov-Stahl theory. We prove the existence of a solution and present the strategy behind its construction.

**Chad Berner**

### Frame-like Fourier Expansions for Finite Borel Measures

We provide examples of finite Borel measures on the torus that are not absolutely continuous or singular but whose $L^2$ spaces admit Fourier frame-like expansions. We will also discuss some properties of these measures. Additionally, we discuss a Fourier frame-like expansion for $L^2$ spaces whose measures are "slice-singular" Borel probability measures on the d dimensional torus. This also allows us to discuss a Fourier frame-like expansion for $L^2$ spaces whose measures are singular Borel probability measures on the real line.

**Shreedhar Bhat**

### $p$-Skwarczyński Distance

In this work, we will discuss some of the new tools to better study the $p$-Bergman space ($A^p$). We also introduce a new distance on a bounded domain using the "minimizer" functions on $A^p(\Omega)$. We discuss its invariance, completeness and other aspects related to it.

**Nick Castillo**

### Global Rational Approximations of Functions with Factorially Divergent Asymptotic Series

Rational approximations of functions offer a rich mathematical theory. Touching subjects such as orthogonal polynomials, potential theory and of course differential equations. In this poster we will present a specific type of rational approximant, factorial expansions. In recent work with O. Costin and R. Costin we have developed a theory of dyadic expansions which improve the domain and rate of convergence when compared to the classical methods found in the literature. These results provide a general method for producing rational approximations of Borel summable series with locally integrable branch points. Surprisingly, these expansions capture the asymptoticly important Stokes phenomena. Additionally, we find applications in operator theory on Hilbert spaces providing new representations for (bounded and unbounded) positive and self-adjoint operators in terms of the semigroups and unitary groups they generate. Finally, as an example of an important application we discuss representing the tritronquée solutions of Painlevé's first equation PI.

**Ana Colovic**

### Garsia norm of Hankel Operators in Weighted Hardy spaces

We discuss the boundedness of Hankel operators between a weighted Hardy space and a weighted $L^2$ space, with two different Muckenhoupt weights. In the Lebesgue measure setting, Hankel operator with a symbol f is bounded if and only if its symbol has a bounded Garsia norm, or equivalently, a bounded BMO norm. We generalize this result to the case of two weights, with the appropriate generalization of the Carleson embedding theorem.

**Abdullah Helal**

### Proper Holomorphic Mappings Between Ball Complements

*Joint work with Achinta Kumar Nandi and Jiří Lebl*

We show that a holomorphic mapping $f : \mathbb{C}^n \setminus \overline{B_n} \to \mathbb{C}^N \setminus \overline{B_N}$ is proper if and only if it is a polynomial proper mapping from $B_n$ to $B_N$. Additionally we seek to classify proper holomorphic mappings $f : B_n \setminus r\overline{B_n} \to B_N \setminus R\overline{B_N}$, for $0 < r,R < 1$. We show that $f$ maps all smaller spheres inside $B_n$ to smaller spheres in $B_N$ if and only if it is a sum of proper homogeneous maps. Moreover, we show that if $f$ is of degree $m$, and if it maps at least $(m-2)$-smaller spheres inside $B_n$ to smaller spheres inside $B_N$, then it must be a sum of proper homogeneous maps, and we construct general monomial examples to show that the limit $(m-2)$ is strict.

**German Mora Saenz**

### Spectral Stability of Traveling Waves in a Thin-Layer Two-Fluid Couette Flow

*Joint work with Saleh Tanveer*

We consider linear stability of traveling waves in a thin-film model for 2-fluid Couette flow when a thin layer of the more viscous fluid resides next to the stationary wall. We prove that in a neighborhood of a bifurcation point from a flat interface, characterized by positive integer $k_b$, the principal branch ($k_b = 1$) is spectrally stable while all other branches ($k_b > 1$) is spectrally unstable. For larger amplitude traveling waves, we establish a number of conditional theorems where the conditions were checked with help of computer assist for a set of parameter values. Using these theorems, we rigorously confirm earlier numerical evidence on stability and instability over a range of parameters.

**Kenta Miyahara**

### Connection Formulae for the Radial Toda Equations

*Joint work with Martin Guest, Alexander Its, Maksim Kosmakov, and Ryosuke Odoi*

Since the invention of the 1D Toda lattice equation in 1967, many types of the Toda equations have been considered. In this presentation, we will focus on a particular case of the 2D periodic Toda equations with radial symmetry. Our goal is to investigate the asymptotic expressions of the solution near singularities and give connection formulae between them.

**Achinta Kumar Nandi**

### Boundary Behavior of Geodesics in Asymptotically Symmetric Spaces

*Joint work with Sean Curry*

We study the boundary behavior of geodesics in asymptotically hyperbolic and asymptotically complex hyperbolic spaces. In the asymptotically hyperbolic case, our argument simply recovers the previous work of Mazzeo but we give a new treatment of this case using a canonical symplectic structure on the cotangent bundle of Mazzeo's 0-cotangent bundle to describe the geodesic flow as a cotangent flow that is sufficiently regular up to the boundary. This approach gives a model for the more complicated situation of asymptotically complex hyperbolic spaces, which we handle in an analogous fashion by introducing and using a canonical symplectic structure on Epstein-Melrose-Mendoza's $\Theta$-cotangent bundle. We conclude by discussing the application of our results to geodesics of canonical metrics on strictly pseudoconvex domains in $\mathbb{C}^n$, $n > 1$.

**Tomas Rodriguez**

### Compactness of Toeplitz operators with Continuous Symbols on Pseudoconvex Domains in $\mathbb{C}^n$

Let $\Omega$ be a bounded pseudoconvex domain in $\mathbb{C}^n$ with Lipschitz boundary and $\phi$ be a continuous function on $\overline{\Omega}$. We show that the Toeplitz operator $T_\phi$ with symbol $\phi$ is compact on the weighted Bergman space if and only if $\phi$ vanishes on the boundary of $\Omega$. We also show that compactness of the Toeplitz operator $T^q_\phi$ on $\overline{\partial}$-closed $(p,q)$-forms for $0 \leq p \leq n$ and $q \geq 1$ is equivalent to $\phi = 0$ on $\overline{\Omega}$.