MWAA 2024

Indiana University Bloomington
October 11–13th

Friday		
15:30–16:00	**Coffee**	
16:00–17:00	**K. Bickel**	Multivariate Bounded Rational Functions
Saturday		
09:00–09:40	**Posters & Coffee**	
09:40–10:20	**A. Christopherson**	Weighted Bergman Kernels, Weak-Type Estimates, and Schur's Test on Non-Smooth Domains in $\mathbb{C}^n$
10:30–11:10	**R. Matzke**	Riesz Energy with an External Field: Dimensionality of Minimizers
11:20–12:00	**L. Hatcher**	A Hot Spots Theorem for the Zaremba Eigenvalue Problem with Small Dirichlet Region
12:00–14:00	**Lunch**	
14:00–14:40	**A. Zhang**	Complex Analytic Approach to Spectral Problems for Differential Operators
14:50–15:30	**V. Mastrantonis**	A Convex-Complex Approach to Mahler's Conjectures
15:30–16:00	**Coffee**	
16:00–16:40	**Ch. Giannitsi**	An Overview of Sparse Domination and Hi-Low Decomposition in the Context of Discrete Harmonic Analysis
16:50–17:30	**L. Kryvonos**	On a Problem of E. Meckes for the Unitary Eigenvalue Process on an Arc
18:30–21:00	**Conference Dinner**	**Samira Restaurant**
Sunday		
09:00–09:40	**Posters & Coffee**	
09:40–10:20	**N. McCleerey**	Asymptotics for some Singular Monge-Ampère Equations
10:30–11:10	**D. Ryou**	Fourier Restriction and Well-Approximable Numbers

Plenary Talks

Kelly Bickel
Bucknell University

Multivariate Bounded Rational Functions

Joint work with Greg Knese, James Pascoe, and Alan Sola

One-variable rational functions q/p that are bounded on a domain U are easy to describe; after cancelling common factors, p cannot have zeros on the closure of U. In contrast, even on nice two-variable domains like the bi-upper half-plane $\mathbb{H}^2$ or the unit bidisk $\mathbb{D}^2$, the multivariate situation is surprisingly complicated. The denominator p of a bounded rational function must still be stable (i.e., have no zeros on the domain), but it can now have boundary zeros. This leads to a host of questions such as:

(1) Given a polynomial p, which numerators q make q/p bounded on the domain (or at least near a given boundary zero of p)?
(2) What types of behaviors do bounded rational functions exhibit near boundary singularities?

In this talk, we will give an overview of recent results related to bounded rational functions, related stable polynomials, and more specific rational inner functions on the poly-upper half-plane and polydisk.

Adam Christopherson
Baylor University

Weighted Bergman Kernels, Weak-Type Estimates, and Schur's Test on Non-Smooth Domains in $\mathbb{C}^n$

Joint work with K. D. Koenig

We will discuss tools for studying the Bergman kernel and projection, a fundamental singular integral operator in complex analysis, on generalized non-smooth domains in $\mathbb{C}^2$ and $\mathbb{C}^3$. To obtain the weak-type regularity and a sharp range of L^p boundedness for the Bergman projection, we use proper holomorphic mappings and apply Schur's test using asymptotic results on the polydisk. In particular, we show that in our non-smooth setting, the Bergman projection satisfies a weak-type estimate at the upper endpoint of L^p boundedness but not at the lower endpoint.

Carrie Clark
University of Illinois Urbana-Champaign

Riesz p-Capacity Properties: Continuity, Diameter, and Volume

Joint work with Richard S. Laugesen
This talk was scheduled for Saturday Oct. 12 but the speaker was unable to appear.

The Riesz p-capacity of a compact set in Euclidean space is defined in terms of an energy optimization problem with pair-wise interaction kernel $|x-y|^p$. In this talk, I will present properties of capacity as a function of p, namely that capacity is left-continuous with respect to p and is right-continuous for sets satisfying an additional hypothesis. Moreover, diameter and volume are recovered in the endpoint limits.

Christina Giannitsi
Vanderbilt University

An Overview of Sparse Domination and Hi-Low Decomposition in the Context of Discrete Harmonic Analysis

This talk is an exposition of certain important techniques in discrete harmonic analysis and is aimed at an audience that is familiar with the material of an average, introductory, graduate harmonic analysis course. We will introduce sparse domination and its significance for maximal inequalities, as well as the circle method and Hi-Low decomposition for obtaining ℓ^p-bounds for operators. The techniques are going to be examined in the context of discrete averaging operators, and we will be referencing joint works with Michael Lacey, Hamed Mousavi, and Yaghoub Rahimi.

Lawford Hatcher
Indiana University Bloomington

A Hot Spots Theorem for the Zaremba Eigenvalue Problem with Small Dirichlet Region

The hot spots conjecture of Rauch states that a second Neumann eigenfunction of the Laplacian on a simply connected domain in Euclidean space has no interior extrema. In the past year or so, several researchers have considered a corresponding question about the first Zaremba (i.e., mixed Dirichlet-Neumann) eigenfunction. We will present a new theorem showing that on convex domains with connected and sufficiently small Dirichlet region, the first mixed eigenfunction indeed has no interior critical points.

Liudmyla Kryvonos
Vanderbilt University

On a Problem of E. Meckes for the Unitary Eigenvalue Process on an Arc

Joint work with Ed Saff

Given a random unitary $n \times n$ matrix and an arc $[0, \theta]$, $0 < \theta < 2\pi$, on the unit circle, we consider an eigenvalue counting function $\mathscr{N}_\theta := \#\{j : 0 < \theta_j < \theta\}$ and explore some of the consequences of the determinantal structure of the eigenvalue processes for $\mathscr{N}_\theta$. Specifically, we study the asymptotics for the eigenvalues of the kernel of the unitary eigenvalue process and relate the question to the following energy problem on the unit circle, which is of independent interest. Namely, for given θ and q, $0 < q < 1$, we determine the function

$$J(q) = \inf\{I(\mu) : \mu \in \mathscr{P}(S^1), \mu(A_\theta) = q\},$$

where $I(\mu) := \iint \log \frac{1}{|z-\zeta|} d\mu(z) d\mu(\zeta)$ is the logarithmic energy of a probability measure μ supported on the unit circle and A_θ is the arc from $e^{-i\theta/2}$ to $e^{i\theta/2}$.

Vlassis Mastrantonis
University of Maryland

A Convex-Complex Approach to Mahler's Conjectures

Joint work with B. Berndtsson and Y. Rubinstein

By applying techniques from convex geometry, we establish sharp bounds on the Bergman kernels of tube domains.

In 2012, Nazarov discovered that the Mahler volume of a convex body is bounded from below by the Bergman kernel of the tube domain over the body. He suggested that finding an optimal lower bound on such Bergman kernels could lead to a resolution of the celebrated Mahler's conjecture (1930's). In 2014, Błocki conjectured that this optimal bound is obtained by a cube. We prove Błocki's conjecture in dimension $n = 2$ using techniques inspired by shadow systems in convex geometry. We also explain why Nazarov's and Błocki's approach is a complex approach to an "L^1-Mahler conjecture" rather than the original Mahler conjecture, and we describe how this gap might be bridged. Lastly, we use symmetrization techniques to obtain sharp upper bounds in all dimensions, establishing Santaló-type inequalities for Bergman kernels.

Ryan W. Matzke
Vanderbilt University

Riesz Energy with an External Field: Dimensionality of Minimizers

Joint work with D. Chafaï, E. Saff, M. Vu, and R. Womersley

We will discuss the minimization of Riesz energies with external fields

$$I_{s,V}(\mu) = \int_{\mathbb{R}^d}\int_{\mathbb{R}^d}\left(\frac{1}{s}\|x-y\|^{-s} + V(x) + V(y)\right)d\mu(x)d\mu(y).$$

We are interested in how the choices of s (the strength of repulsion between "electrons") and V (the external field) affect the structure of the equilibrium measure, particularly the dimension of its support. We will focus on radially symmetric external fields of the form $V(x) = \gamma\|x\|^{\alpha}$ for $\alpha, \gamma > 0$ (these act as an attractive sink at the origin), and will classify exactly when the support is the uniform measure on a sphere.

Nicholas McCleerey
Purdue University

Asymptotics for some Singular Monge-Ampère Equations

Given a psh function u in the Cegrell class and a smooth, non-negative function g, it is known that one can always solve the Monge-Ampère equation $MA(u_g) = g^n MA(u)$, with some form of Dirichlet boundary values, by work of Åhag-Cegrell-Czyż-Pham. Left unsaid in their work is how u_g compares with u near the polar set $u = -\infty$. We present a simple condition on u which allows us to show that u_g behaves (to leading order) like gu away from the boundary of $g > 0$. Our results also apply to complex Hessian equations.

Donggeun Ryou
Indiana University Bloomington

Fourier Restriction and Well-Approximable Numbers

Joint work with Robert Fraser and Kyle Hambrook

Suppose that μ is a Borel probability measure on $\mathbb{R}^d$ such that $\mu(B(x,r)) \lesssim r^a$ for all $x \in \mathbb{R}^d$ and all $r > 0$ and $|\widehat{\mu}(\xi)| \lesssim (1+|\xi|)^{-b/2}$ for all $\xi \in \mathbb{R}^d$. The Mockenhaupt-Mitsis-Bak-Seeger Fourier restriction theorem says that for each $p \geq (4d-4a+2b)/b$,

$$\|\widehat{f\mu}\|_{L^p(\mathbb{R}^d)} \lesssim_p \|f\|_{L^2(\mu)}$$

holds for all $f \in L^2(\mu)$. We use a deterministic construction to prove the optimality of range of p in the Mockenhaupt-Mitsis-Bak-Seeger Fourier restriction theorem for dimension $d = 1$ and parameter range $0 < a, b \leq d$ and $b \leq 2a$. Previous constructions by Hambrook-Łaba and Chen required randomness and only covered the range $0 < b \leq a \leq d = 1$.

Ashley Zhang
Vanderbilt University

Complex Analytic Approach to Spectral Problems for Differential Operators

Joint work with Alexei Poltoratski

This talk will be about applications of complex function theory to spectral problems for canonical systems, which constitute a broad class of second order differential equations. I will start with the basics of Krein-de Branges theory. Then I will present an explicit algorithm for inverse spectral problems developed by Makarov and Poltoratski for locally-finite periodic spectral measures, as well as an extension of their work to certain classes of non-periodic spectral measures. Finally, I will talk about some recent developments on direct spectral problems.

Poster Presentations

Chad Berner

Operator Orbit Frames and Frame-like Expansions

Frames in a Hilbert space that are generated by operator orbits are vastly studied because of the applications in dynamic sampling and signal recovery. Furthermore, it is known that the Kaczmarz algorithm for stationary sequences in Hilbert spaces generates a frame that arises from an operator orbit. We discuss that every frame generated by a not surjective operator in any Hilbert space arises from the Kaczmarz algorithm. Furthermore, we note that the operators generating these frames are similar to rank one perturbations of unitary operators. Finally, we describe a large class of operator orbit frames that arise from Fourier expansions for singular measures.

Abdullah Helal

Rational m-fold Sphere Maps

Joint work with Achinta Kumar Nandi and Jiří Lebl

We introduce and study rational m-fold sphere maps, that is, rational maps taking m spheres to m spheres. We show that a polynomial m-fold sphere map of degree m or less is an ∞-fold sphere map, that is, takes infinitely many spheres to spheres. Similarly, every rational m-fold sphere map of degree less than m is an ∞-fold sphere map. We then prove that ∞-fold sphere maps are, up to a unitary transformation, direct sums of finitely many homogeneous sphere maps.

Tomas Lasic Latimer

The Riemann-Hilbert Problem and its Applications to Differential and Difference Equations

The poster describes the origins of the Riemann-Hilbert Problem and a recent application in determining the asymptotic behavior of a class of discrete orthogonal polynomials.

Kenta Miyahara

Asymptotics of Real Solutions of sinh-Gordon Equation

Joint work with Alexander Its and Maxim Yattselev

We consider the asymptotic behavior of the real-valued solutions $u(x)$ of the sinh-Gordon reduction of the Painlevé III equation ($\mathrm{P}_{\mathrm{III}}^{\mathrm{shG}}$ in short) on the real line as $x \to 0^+$ and $x \to +\infty$. Our methodology is to solve the associated Riemann-Hilbert problem with $\mathrm{P}_{\mathrm{III}}^{\mathrm{shG}}$ using the nonlinear steepest descent method of Deift and Zhou. In this way, we obtain the desired asymptotics along with the asymptotic locations of the poles of $\mathrm{P}_{\mathrm{III}}^{\mathrm{shG}}$ accumulating at 0 and ∞.

Andres Quintero Santander

On Stationary Measures for a Family Generated by Hénon-type Maps

Joint work with Roland Roeder and Julio Rebelo

For a classical Hénon $h(x,y) = (x^2 + c - ay, x)$ we consider the map $g := \varphi^{-1} \circ h\varphi$, where φ is a rotation by $\pi/4$, our main result is showing that the group G generated by h and g is the free group on two generators and that any stationary measure for the measure $\nu = 1/4\delta_h + 1/4\delta_g + 1/4\delta_{h^{-1}} + 1/4\delta_{g^{-1}}$ on G must have compact support. We explore questions and possible generalizations of this result.